

Twistors and 2T-physics¹

Itzhak Bars

Department of Physics and Astronomy
University of Southern California
Los Angeles, CA 90089-0484

ABSTRACT

Two-Time physics applies broadly to the formulation of physics and correctly describes the physical world as we know it. Recently it was applied to a 2T reformulation of the $d = 4$ twistor superstring, which was suggested by Witten as an efficient approach for computations of physical processes in the maximally supersymmetric $N = 4$ Yang-Mills field theory in four dimensions. The 2T formalism provides a six dimensional view of this theory and suggests the existence of other $d = 4$ dual forms of the same theory. Furthermore the 2T approach led to the first formulation of a twistor superstring in $d = 10$ appropriate for $\text{AdS}_5 \times \text{S}^5$ backgrounds, and a twistor superstring in $d = 6$ related to the little understood superconformal theory in $d = 6$. The proper generalization of twistors to higher dimensions is an essential ingredient which is provided naturally by 2T-physics. These developments are summarized in this lecture².

1 Introduction

Two-Time Physics (2T-physics) is a natural framework in higher spacetime with 2T signature that encodes and unifies many aspects of physics, from simple quantum mechanics to strings [1]. The 2T-physics formalism is free from any problems with unitarity or causality, thanks to appropriate gauge symmetries. One-time Physics (1T-physics) is correctly embedded in the 2T-physics framework.

The 1T interpretation of a 2T-physics system depends on the *perspective* of embedding phase space in $(d - 1, 1)$ dimensions into phase space in $(d, 2)$ dimensions. This is done by making a gauge choice, which yields a holographic image of the physical subspace of the $(d, 2)$ phase space. Consequently, the same system in $(d, 2)$ is viewed as various holographic dynamical images in various $(d - 1, 1)$ embeddings. From the point of view of 2T-physics, many aspects of 1T-physics, such as the Hamiltonian

¹Research supported by the US Department of Energy, Grant No. DE-FG03-84ER40168

²Lectures delivered at “Twistor String Theory”, Oxford, England, Jan. 2005; and “Fundamental Interactions and Twistor Methods”, Wroclaw, Poland, Oct. 2004. Transparencies available at <http://physics.usc.edu/~bars/papers/twistor.pdf>.

with interactions, *time and space, are all emergent concepts* that depend on the embedding. In particular *twistors* provide a particular holographic image of the system in $(d, 2)$. The 2T-physics formalism leads to the proper generalization of twistors to any dimension [2] as will be outlined in this lecture and presented in more detail in [3].

The $d = 4$ twistor superstring developed by Witten [4, 5] and Berkovits [6, 7, 8] coincide with a 2T superstring [9] in $(4, 2)$ dimensions with $SU(2, 2|4)$ supersymmetry, when the 2T superstring is discussed from the perspective of twistors. In addition, the 2T superstring approach in $(6, 2)$ dimensions with $OSp(8|2)$ supersymmetry, and $(10, 2)$ dimensions with $SU(2, 2|4)$ supersymmetry, yielded new twistor superstrings that were conceived for the first time, thus demonstrating the usefulness of the 2T-physics formalism. The $(10, 2)$ case yields a holographic twistor description of the space $AdS_5 \times S^5$, with a twistor superstring whose particle limit spectrum is the full Kaluza-Klein towers of type IIB supergravity compactified on $AdS_5 \times S^5$. This spectrum contains information about hidden dimensions with $(10, 2)$ signature as discussed earlier [10][11], and the new superstring extends the $(10, 2)$ view of $AdS_5 \times S^5$ to the realm of strings. Similarly, the $(6, 2)$ case yields a new twistor superstring whose particle limit describes a supermultiplet of a peculiar self-dual superconformal theory in $(5, 1)$ dimensions whose physical space (in a lightcone gauge) consists of 8 bosons and 8 fermions. Another description [9] of this supermultiplet is the unitary representation of $OSp(8|4)$ in the oscillator formalism [12] which coincides with the $OSp(8|4)$ doubleton given in [13]. This six dimensional conformal theory is expected to exist as an interacting theory, but it cannot be described in the form of a field theory [14]. The twistor superstring may be a possible description of this interacting theory.

Perhaps I should give some of the history that motivated the development of 2T-physics. It is often stated that 32 supersymmetries is the maximum possible number of supersymmetries, and therefore 11 dimensions, which has a spinor of 32 real components, is the maximum number of dimensions in a supersymmetric theory of fundamental physics. However, the Weyl spinor in 12 dimensions, with signature $(10, 2)$, also has a real spinor with 32 components. Furthermore, the maximally extended supersymmetry algebra, called the M-algebra, has a symmetry of isomorphisms that include $SO(10, 2)$, which can be interpreted as acting on a 12-dimensional spacetime with signature $(10, 2)$. This point of view was expressed for the first time in 1995 and related to dualities in one of my talks [15] and later further developed in [16]. The possibility of hidden timelike dimensions in M-theory was strengthened further by the hidden symmetry structures in the web of dualities involving D-branes, as in F-theory [17] and S-theory [16].

Of course hints coming from symmetries, although suggestive, are not enough to infer extra spacetime dimensions. However, a dynamical theory involving the higher spacetime, which describes the recognizable world, would go a long way toward understanding the higher spacetime. This was the motivation behind the development of 2T-physics, which after some attempts [18] finally took the correct physical form starting in 1998 as described in [1]. The backbone of the 2T structure is an $Sp(2, R)$ gauge symmetry that acts in phase space. By generalizing this symmetry in several

appropriate ways 2T-physics makes contact with the real world. By now it is abundantly clear that the 2T framework describes correctly simple everyday physics as well as complicated structures in string theory.

We don't have to wait until we discover the correct formulation of M-theory to know that 2T-physics is correct, and that it teaches us that there is a sense in which $(d, 2)$ dimensions provide a higher unifying framework. This view is already born out in simple classical and quantum mechanical systems, and there are useful non-trivial consequences that follow from it. We have now come back full circle to using 2T-physics techniques to try to construct corners of M-theory, such as the twistor superstrings given in [9] and described briefly in this lecture. 2T-physics suggests that we should look for a formulation of M-theory in $(11, 2)$ dimensions with global symmetry $\text{OSp}(1|64)$ [16][19][20].

In this lecture I will first give a brief description of the concepts in 2T-physics, and then describe the twistors and the twistor superstrings in $d = 3, 4, 5, 6, 10$ that were constructed by using the formalism. The twistors that emerge in the new twistor superstrings can be discussed without the full 2T formalism, as will be done in part of this lecture and in [3], so this aspect can be carried away and applied usefully elsewhere without the need for the full 2T package. However, the full 2T formalism is what provides the easy proof that the new twistors in the higher dimensions describe $\text{AdS}_5 \times S^5$ (for $d = 10$), the six dimensional conformal theory (for $d = 6$) respectively, and of course the Super Yang Mills (SYM) theory (for $d = 4$). The 2T version of the theory is far richer because it relates the twistors to other dual forms of the same theory, and this aspect may be crucial ultimately for deeper understanding and for practical progress.

2 2T-physics

2.1 Gauge symmetry is the origin of spacetime signature

First I suggest the point of view that gauge symmetry is at the origin of the 1T spacetime signature $(-, +, \dots, +)$, and then show that the same point of view leads to the 2T signature $(-, -, +, \dots, +)$.

Consider the action of a particle on the worldline which is invariant under τ reparametrizations

$$S = \int_1^2 d\tau (\partial_\tau x^\mu p_\mu - e(\tau) Q(x, p)), \quad (1)$$

where $e(\tau)$ is the gauge field that transforms as $\delta_\epsilon e(\tau) = \partial_\tau \epsilon(\tau)$, while the infinitesimal transformations of $x(\tau), p(\tau)$ are given by the Poisson brackets $\delta_\epsilon x^\mu = \epsilon(\tau) \{Q, x^\mu\} = -\epsilon(\tau) \partial Q / \partial p_\mu$ and $\delta p_\mu = \epsilon(\tau) \{Q, p_\mu\} = \epsilon(\tau) \partial Q / \partial x^\mu$. The Lagrangian transforms into a total derivative so that $\delta_\epsilon S = \int_1^2 d\tau \partial_\tau (\epsilon Q) = 0$, with boundary conditions $\epsilon(\tau_1) = \epsilon(\tau_2) = 0$. The well known free massless relativistic particle action corresponds to $Q(x, p) = \frac{1}{2} \eta^{\mu\nu} p_\mu p_\nu$, while the general $Q(x, p)$ can describe all possible interactions of the particle in any background. For example for

an electromagnetic background we have $Q = \frac{1}{2}\eta^{\mu\nu} (p_\mu - qA_\mu(x))(p_\nu - qA_\nu(x))$. Evidently Q is the generator of the local gauge symmetry. The equation of motion for e requires $Q(x, p) = 0$. The space in which the gauge symmetry generator vanishes is evidently gauge invariant (a singlet under gauge transformations). Therefore, this equation is interpreted to mean that the physical space (either classical or quantum), defined to be the solution space of $Q(x, p) = 0$, is gauge invariant.

Consider at first the simplest case $Q(x, p) = p^2 = 0$. We notice that if the signature in $p^2 = \eta^{\mu\nu} p_\mu p_\nu$ is Euclidean the only solution of $p^2 = 0$ is $p_\mu = 0$, so that no non-trivial solution exists for physical space for Euclidean signature. To describe non-trivial motion, target space-time must have 1 time

$$p \cdot p = -p_0^2 + \vec{p}^2 = 0.$$

There are nontrivial solutions also with more timelike dimensions, however τ reparametrization is insufficient to remove the ghosts of more than 1 timelike dimension. Therefore unitarity of the theory requires that spacetime cannot have more than 1 time. Thus, τ reparametrization requires just one time coordinate no more and no less. Causality corresponds to admitting only nonwinding maps $\tau \rightarrow x^\mu(\tau)$.

From the simplest case $Q(x, p) = p^2$ we have learned that the signature of the parameter $\varepsilon(\tau)$ in τ reparametrization is timelike. Thus, a timelike (or lightlike, but not spacelike) degree of freedom can be removed from $x^\mu(\tau)$ and similarly a timelike degree of freedom can be removed from p^μ by solving the constraint $p^2 = 0$. For the more general $Q(x, p)$ the signature of $\varepsilon(\tau)$ is the same as before, therefore the gauge symmetry will remove a timelike degree of freedom, not a spacelike one, and the constraint $Q(x, p) = 0$ can have a solution provided target spacetime has signature $(-, +, \dots, +)$ that includes a timelike degree of freedom. Therefore we deduce that the gauge symmetry requires that there has to be one timelike degree of freedom in any target spacetime (relativistic, nonrelativistic, curved, etc.).

This reasoning is broadened by starting with a worldline action that is invariant under $\text{Sp}(2, R)$ gauge symmetry introduced in [1]. For the simplest case $\text{Sp}(2, R)$ acts on phase space as a doublet, and an invariant action is written as follows

$$\begin{aligned} \text{Sp}(2, R) \text{ doublet: } \begin{pmatrix} X^M(\tau) \\ P^M(\tau) \end{pmatrix} &\equiv X_i^M \ i = 1, 2 \\ A_i^j \text{ gauge field: } D_\tau X_i^M &= \partial_\tau X_i^M - A_i^j X_j^M, \\ S &= \frac{\eta_{MN}}{2} \int d\tau (\varepsilon^{ij} \partial_\tau X_i^M X_j^N - A^{ij} X_i^M X_j^N) \\ \text{Sp}(2, R) \text{ generators : } X \cdot X, X \cdot P, P \cdot P, &\rightarrow X_i \cdot X_j = 0 \end{aligned} \tag{2}$$

We deduce that physical space must be $\text{Sp}(2, R)$ singlet $X_i \cdot X_j = 0$, and then ask for which signature η^{MN} can we find a nontrivial physical space? We quickly learn that there is no non-trivial content for zero times or one time, and therefore we must admit that target space-time *must* have 2 times

$$-X_{0'}^2 - X_0^2 + X_I^2 = 0, \text{ etc. } X \cdot P = P \cdot P = 0$$

Compared to τ reparametrization, $\text{Sp}(2, R)$ has 2 more gauge symmetries and 2 more constraints. These eliminate 2 more degrees of freedom from both X^M and P^M . Thus by starting from a space with signature $(d, 2)$ we end up with an emergent spacetime with signature $(d - 1, 1)$ by making various gauge choices

$$(d, 2) - (1, 1)_{\text{signature of extra gauge parameters}} = (d - 1, 1)_{\text{emergent space-time}}$$

We conclude that physical spacetime has $(d - 1)$ space and 1 time, just like before, but these *must be embedded in a higher spacetime* with signature $(d, 2)$.

This would not be very deep if there were a single solution to this embedding. The non-trivial aspect is that there are many ways in which phase space in $(d - 1, 1)$ is embedded in phase space in $(d, 2)$, and this provides many ways in which time (or Hamiltonian) is defined in the emergent spacetime. The embedding provides a holographic image of the events and motion in the $(d, 2)$ space, which can be interpreted very differently from the perspective of each of the emergent spaces in $(d - 1, 1)$ since each such space defines time (and Hamiltonian) differently than one another. Even though we start from a single well defined 2T-physics system in $(d, 2)$, we end up with many holographic pictures that are interpreted differently, with different Hamiltonians, in 1T-physics [21].

Considering also unitarity we find that no more than 2 times are possible since the $\text{Sp}(2, R)$ gauge symmetry cannot remove the ghosts from a spacetime with more time-like dimensions. In this case causality is satisfied since the situation in the emergent $(d - 1, 1)$ is no different than the one time situation.

The simple model above has been generalized to include arbitrary interactions with all possible background fields [22]. The generalized action is $S = \int d\tau (\partial_\tau X^M P_M - \frac{1}{2} A^{ij} Q_{ij}(X, P))$ and the gauge symmetry is still $\text{Sp}(2, R)$, with $\delta A_i^j = \partial_\tau \omega_i^j + [A, \omega]_i^j$ and Q_{ij} the generator for infinitesimal transformations $\delta X^M = \omega^{ij}(\tau) \{Q_{ij}, X^M\} = \omega^{ij}(\tau) \partial Q_{ij} / \partial P_M$, and $\delta P^M = \omega^{ij}(\tau) \{Q_{ij}, P^M\} = -\omega^{ij}(\tau) \partial Q_{ij} / \partial X^M$. The generalized $Q_{ij}(X, P)$ depend on background fields in $(d, 2)$ dimensions, such as $A_M(X)$, $G_{MN}(X)$, etc., and those are constrained by the requirement that $Q_{ij}(X, P)$ must satisfy the $\text{Sp}(2, R)$ algebra. This leads to differential equations for the background fields. All possible solutions are obtained in [22], and it is shown that this covers all possible interactions with backgrounds in 1T-physics, including the Maxwell field $A_\mu(x)$, the gravitational field $g_{\mu\nu}(x)$, etc. as described in Eq.(1). A similar but less complete analysis has been done also for spinning particles [26].

In this way we can argue that possibly all of 1T-physics can be embedded in 2T-physics. It is evident that from the same 2T-physics model, with fixed backgrounds, one can obtain in principle many 1T-physics systems in the form of various holographic images, thus showing that 2T-physics unifies the various dynamics in 1T into a parent theory in 2T that reveals the hidden relationships and symmetries that are not evident at all in the 1T approach.

2.2 Some examples of emergent dynamics and spacetimes

Consider the simplest case of a 2T-physics action for a particle in flat $(d, 2)$ spacetime as given in Eq. (2). In this lecture I will illustrate two solutions of the constraints

that are inequivalent from the point of view of 1T-physics, but which are evidently equivalent under the $\text{Sp}(2, R)$ gauge transformations of the 2T theory, and therefore dual to one another. The first case is the relativistic massless particle and the second case is the hydrogen atom. Sometimes it will be convenient to express the flat $(d, 2)$ metric in lightcone type basis. By using the extra dimensions $X^{0'}, X^{1'}$ we define $X^{\pm'} = (X^{0'} \pm X^{1'})/\sqrt{2}$ so that the metric is $ds^2 = -(dX^{0'})^2 + (dX^{1'})^2 + dX^\mu dX^\nu \eta_{\mu\nu} = -2dX^{+'}dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$ with $\eta_{\mu\nu}$ the $(d-1, 1)$ Minkowski metric.

2.2.1 Relativistic spacetime gauge

In the lightcone type basis we choose two gauges by fixing $X^{+'}(\tau) = 1$, $P^{+'}(\tau) = 0$ for all τ , and then solve two of the constraints $X^2 = X \cdot P = 0$. This solution is given by

$$\begin{aligned} X^M &= \begin{pmatrix} +', x^{\bar{'}}/2, x^\mu \end{pmatrix} & X \cdot X &= -2X^{-'}X^{+'} + X^\mu X^\nu \eta_{\mu\nu} = 0 \\ P^M &= \begin{pmatrix} +', x^{\bar{'}} \cdot p, p^\mu \end{pmatrix} & X \cdot P &= 0, \quad P \cdot P = p^2 \end{aligned} \quad (3)$$

There remains one more gauge choice to be made and one more constraint $p^2 = 0$ to be solved, but we refrain from doing those steps for now. To interpret our choice of independent variables x^μ, p^μ we investigate the form of the gauge fixed action

$$\text{gauge fixed } S = \int d\tau \left(\dot{x} \cdot p - \frac{1}{2} A^{22} p^2 \right)$$

and note that x^μ, p^μ are canonical variables which describe the massless relativistic particle in $(d-1, 1)$ dimensions. To be sure of this fact, we can also investigate that the original equations of motion $\dot{X}^M = A^{22} P^M + A^{12} X^M$ and $\dot{P}^M = -A^{11} X^M - A^{12} P^M$ are fully consistent with the equations of motion that follow from the gauge fixed action.

The original gauge invariant action was symmetric under the $\text{SO}(d, 2)$ global symmetry. The conserved gauge invariant generators of that symmetry are $L^{MN} = \varepsilon^{ij} X_i^M X_j^N = X^M P^N - X^N P^M$. Since both the action and the generators are gauge invariant, the gauge fixed action must have a hidden $\text{SO}(d, 2)$ symmetry, with generators given by the gauge fixed form of L^{MN} . Indeed, the gauge fixed generators become the conformal symmetry of the massless particle

gauge fixed L^{MN} become conformal $\text{SO}(d, 2)$

$$\begin{aligned} L^{\mu\nu} &= x^{[\mu} p^{\nu]}, \quad L^{+'-'} = x \cdot p, \\ L^{+' \mu} &= p^\mu, \quad L^{-' \mu} = \frac{x^2}{2} p^\mu - x \cdot p x^\mu. \end{aligned}$$

When the system is quantized in terms of the relativistic variables x^μ, p^μ the L^{MN} must be carefully quantum ordered. The ordering must insure Hermitian L^{MN} relative

to a relativistic norm for the quantum states. When this is done [1] one can compute the Casimir eigenvalue of the $SO(d, 2)$ representation that describes the massless spinless particle. The result is in complete agreement with covariant quantization of the 2T system, and is given by

$$\text{After quantum ordering: } C_2 = \frac{1}{2} L^{MN} L_{MN} = 1 - \frac{d^2}{4}$$

same as $SO(d, 2)$ covariant quantization in $Sp(2, R)$ invariant space

This representation is known as the singleton in $d = 3, 4$ and thus we will call it the singleton for any d . Note that at the classical level (not watching orders of operators) one obtains zero for the Casimir since $L^{MN} L_{MN}$ is constructed from $X^2, P^2, X \cdot P$ which vanish in the physical sector.

2.2.2 H-atom gauge

Another solution of the constraints $X^2 = P^2 = X \cdot P = 0$, is [21]

$$\begin{aligned} X^M &= [r \cos u, \quad r \sin u, \quad \frac{\vec{r} \cdot \vec{p}}{-\alpha} r \sqrt{-2H}, (\vec{r}^i - \frac{\vec{r} \cdot \vec{p}}{\alpha/r} \vec{p}^i)] \quad r \equiv |\vec{r}| \\ P^M &= [\frac{-\alpha \sin u}{r}, \frac{\alpha \cos u}{r}, \left(\frac{\alpha}{r} - \vec{p}^2\right), \sqrt{-2H} \vec{p}^i] (-2H)^{-1/2} \end{aligned} \quad (4)$$

where $H = \frac{\vec{p}^2}{2} - \frac{\alpha}{r}$, and $u = (\vec{r} \cdot \vec{p} - 2\tau H) \frac{\sqrt{-2H}}{\alpha}$. The interpretation of the emergent dynamics is found by examining the gauge fixed action

$$\text{gauge fixed } S = \int d\tau (\partial_\tau \vec{r} \cdot \vec{p} - H) \leftrightarrow \int d\tau \left(\frac{1}{2} (\partial_\tau \vec{r})^2 + \frac{\alpha}{r} \right)$$

Evidently, this is the spinless Hydrogen atom (or a planetary system, etc.). The original $SO(d, 2)$ global symmetry $L^{MN} = X^M P^N - X^N P^M$ must be a hidden symmetry of this action. The gauge fixed generators are (at the classical level)

$$\begin{aligned} L^{00} &= \frac{\alpha}{\sqrt{-2H}}, \quad L^{ij} = r^i p^j - r^j p^i, \quad L^{1'i} = \frac{1}{\sqrt{-2H}} \left(r \cdot p p^i - r^i p^2 - \alpha \frac{\mathbf{r}^i}{r} \right), \\ L^{0'1'} &= -r \cdot p \sin u + \frac{\alpha}{\sqrt{-2H}} \left(1 - \frac{r \mathbf{p}^2}{\alpha} \right) \cos u, \quad L^{01'} = r \cdot p \cos u + \frac{\alpha}{\sqrt{-2H}} \left(1 - \frac{r \mathbf{p}^2}{\alpha} \right) \sin u, \\ L^{0'i} &= r p^i \cos u + \frac{\alpha}{\sqrt{-2H}} \left(\frac{\mathbf{r}^i}{r} - \frac{\mathbf{r} \cdot \mathbf{p}}{\alpha} p^i \right) \sin u, \quad L^{0i} = r p^i \sin u - \frac{\alpha}{\sqrt{-2H}} \left(\frac{\mathbf{r}^i}{r} - \frac{\mathbf{r} \cdot \mathbf{p}}{\alpha} p^i \right) \cos u \end{aligned}$$

In the first line one can recognize the angular momentum and the Runge-Lenz vector that are long known to be conserved quantities of the H-atom system (i.e. commute with H) and that they correspond to a hidden $SO(d)$ symmetry (better known as $SU(2) \times SU(2) = SO(4)$ in 3 space dimensions). However, 2T-physics gives a stronger symmetry, not of the Hamiltonian, but of the action. According to 2T-physics the H-atom action is invariant under $SO(d, 2)$. Indeed this symmetry can be verified

directly³. Before this was understood in 2T-physics no-one seems to have been aware of the symmetry of the action, although there has been discussions of a dynamical $SO(4, 2)$ algebra of the H-atom system.

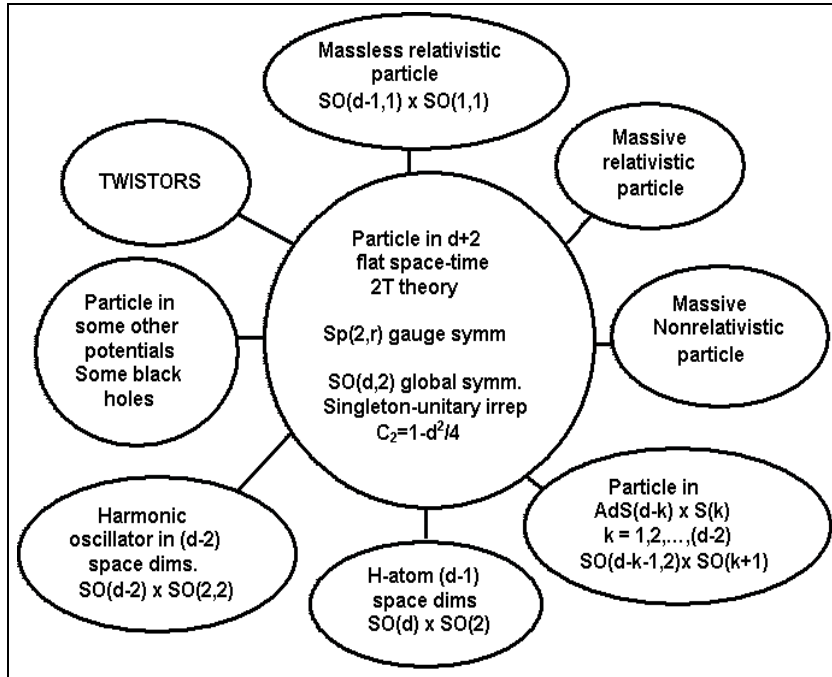
We can go further by quantizing the system, ordering properly the operators and computing the Casimir operator. We find again that C_2 reduces to a pure number which corresponds to the singleton representation [21]

$$\text{After quantum ordering: } C_2 = \frac{1}{2} L^{MN} L_{MN} = 1 - \frac{d^2}{4}$$

This is what it should be according to the general properties of 2T-physics. Indeed L_{MN} are gauge invariant and they should give the same Casimir in any gauge. This also fits the idea of a duality between the free relativistic particle and the H-atom, since these are derived from the same 2T action by gauge fixing, and therefore they can be transformed to each other by the $Sp(2, R)$ gauge transformations. Such $Sp(2, R)$ transformations are easily constructed classically between the gauges (3) and (4). This is expected to succeed also at the quantum level in the form of unitary transformations among dual bases, since the quantum states in either gauge belong to the same representation of $SO(d, 2)$ with the same Casimir operators.

2.2.3 More examples of emergent dynamics/spacetimes

Many more 1T dynamical systems emerge holographically from the same 2T theory given in Eq. (2). The diagram below illustrates some of the cases that have been investigated [21].



$d+2$ to d holography gives many 1T systems.

Each one is a basis for $C_2=1-d^2/4$ irrep of $SO(d,2)$.

³See a homework problem and its solution at <http://physics.usc.edu/~bars/papers/Hatom.pdf>.

These include interacting as well as free systems. The quantum theory has been investigated, and the quantum ordering of the L^{MN} operators has been obtained for the cases of the massless relativistic particle, H-atom, harmonic oscillator, particle on $\text{AdS}_{d-k} \times S^k$ background, particle in the $\text{SL}(3, R)$ black hole, and twistors. In each case it is shown that $C_2 = 1 - d^2/4$.

The 1T systems are derived from the same 2T system by making an $\text{Sp}(2, R)$ gauge choice. In each case some combination of $X^M(\tau), P^M(\tau)$ is gauge fixed to be τ . The canonical conjugate to that choice is always the Hamiltonian written as a function of the remaining phase space degrees of freedom. Although the Hamiltonian (time) looks very different in each case, it still represents a holographic image of the original 2T particle. Thus, each one of these systems represents the same 2T theory although they each have a different interpretation in 1T-physics. In the 1T context they must be interpreted as being dual to each other. In the quantized version each one provides a basis for the same singleton representation of $\text{SO}(d, 2)$. Within the same representation they must correspond to different bases (which diagonalize the respective Hamiltonian) related by unitary transformations. The existence of such relationships are not at all evident in the 1T approach.

Further generalizations include 2T formulations of spinning particles [23], space-time supersymmetry [24], twistors [2][9][3], and some study of 2T-physics in the context of field theory [26] and string theory [25][9]. A lot more basic research is awaiting to be developed in 2T-physics.

With what we know so far about 2T-physics, it is evident that it applies broadly to physics and correctly describes the physical world as we know it. The advantage of 2T-physics over 1T-physics is its unification of various 1T systems into a single 2T system, thus providing a more unified perspective. This aspect could be illuminating for the physics that we already understand in the 1T formalism by taking advantage of the revealed hidden symmetries and by exploring the unsuspected duality type relationships among various 1T dynamical systems. Knowing these facts should shed light on the solution and interpretation of physical systems. Furthermore, the 2T approach could be used as a tool for formulating the physics we don't fully understand yet, such as M-theory. The mystery of M-theory has been my main motivation so far in pursuing and developing this approach.

3 Twistors as a gauge choice in 2T-physics

Twistors were obtained as one of the possible gauge choices in 2T-physics in [2], and this has been further explored in [9] and [3]. For this purpose we consider a group or supergroup G that contains $\text{SO}(d, 2)$ as a subgroup ($G = \text{SO}(d, 2)$ is the smallest choice). A group element $g(\tau) \in G$ is introduced as a degree of freedom in addition to $X^M(\tau), P^M(\tau)$, and it is taken in the smallest representation of G such that $\text{SO}(d, 2) \in G$ is the spinor representation. When G is a supergroup it contains spacetime fermionic spinors that will be useful for spacetime supersymmetric theories. The group element g contains also more bosons beyond (X, P) which, in most cases, can be gauged away by additional gauge symmetries.

g is taken as a singlet under the $\text{Sp}(2, R)$ gauge group, while (X^M, P^M) form a doublet. Next we introduce a further gauge symmetry embedded in G that acts on the *left side of* g , as well as on X^M, P^M as specified below, to have the *correct number of physical degrees of freedom* for a (spinning) particle or superparticle after gauge degrees of freedom are eliminated. On the right side of g we maintain a full global symmetry G , therefore rows of g transform as spinors under the $\text{SO}(d, 2) \in G$. This is where twistors in the spinor representation of $\text{SO}(d, 2)$ will come from.

In most applications G is a supergroup, and the fermions in g are used to supersymmetrize the 2T system. However, it is also possible to discuss purely bosonic cases and even specialize to $G = \text{Spin}(d, 2)$. In this setting it is possible to choose gauges to eliminate degrees of freedom from g and/or from X, P . If all of g is eliminated, as in the purely bosonic $G = \text{Spin}(d, 2)$ case, we remain only with X, P which give the 2T system discussed in the previous sections. However, if all of X, P and some of the g is eliminated we remain with some of the degrees of freedom of g which describe the same 2T system in terms of twistors. If G is an appropriate supergroup that yields the superparticle in one gauge, then in the twistor gauge (with X, P completely eliminated) we obtain the *supertwistor* description of the superparticle. In this way we derive the correct supertwistor representation of several systems of interest in several dimensions as given in [2][9][3] and briefly described here.

3.1 Supersymmetric 2T-physics

The most general case studied corresponds to the supersymmetrization of the spinning particle of spin $n/2$. For the spin= $n/2$ generalization we introduce n fermions $\psi_1^M, \dots, \psi_n^M$ in addition to X^M, P^M, g . Although in this lecture we will mainly discuss the $n = 0$ case (i.e. only X, P, g), we first give the more general Lagrangian

$$L = \frac{\eta^{MN}}{2} (q^{ab} \partial_\tau Y_a^M Y_b^N - A^{ab} Y_{[a}^M Y_{b]}^N) \quad \text{local OSp}(n|2)_{q^{ab} = \text{metric}} \\ - \frac{1}{2[d/2]} (L^{MN} + S^{MN}) \text{Str} (\Gamma^{MN} \partial_\tau g g^{-1}) \quad , \quad g \in G_d \text{ supergroup} \quad (5)$$

For $n = 0$ the first line is the same as Eq. (2) which describes the scalar 2T particle. The second line generalizes it with spacetime supersymmetry. In a specific gauge this system yields the standard superparticle as one of the holographic images [24][2]. The nonzero spin case with supersymmetry has not appeared yet in the literature, it will be discussed in [3], here we provide a brief description.

The first line in Eq. (5) by itself describes the spinning particle with spin $n/2$. This generalizes the scalar 2T-particle in Eq. (2) by replacing the $\text{Sp}(2)$ doublet (X^M, P^M) by the $\text{OSp}(n|2)$ fundamental representation with n fermions, and introducing the corresponding gauge fields A_a^b (which include the $\text{Sp}(2, R)$ gauge fields along with $\text{SO}(d)$ and fermionic counterparts)

$$Y_a = (X^M, P^M, \psi_1^M, \dots, \psi_n^M), \quad D_\tau Y_a^M = \partial_\tau Y_a^M - A_a^b Y_b^M, \quad J^{MN} = q^{ab} Y_a^M Y_b^N = L^{MN} + S^{MN}, \\ \text{fundamental of OSp}(n|2) \quad \text{OSp}(n|2) \text{ gauge field} \quad S^{MN} = \psi_i^{[M} \psi_i^{N]} \quad \text{spin} = \frac{n}{2} \quad (6)$$

This system (without the supersymmetrization of the second line) was discussed in detail in [23]. Since we don't have time to discuss it here, we will specialize to only $n = 0$ in this lecture.

The second line in Eq. (5) corresponds to supersymmetrizing the system in the first line with or without spin. Here the supergroup G_d with N supersymmetries is taken for various dimensions d as one of the following supergroups

$$G_d = \text{OSp}(N|4)_3, \text{SU}(2, 2|N)_4, \text{F}(4)_5, \text{OSp}(8|N)_6, \text{PSU}(2, 2|4)_{10}$$

When $d = 3, 4, 5, 6$ the spinor representation of $\text{SO}(d, 2)$ corresponds to the first block in the fundamental matrix representation of G_d as shown below

$$g(\tau) = \exp \left(\begin{array}{c|c} \frac{1}{4} \Gamma^{MN} \omega_{MN} & \Theta_{\text{spinor}}^{i=1 \dots N} \\ \hline \Theta & R^a \omega_a \end{array} \right) \quad (7)$$

$G_d = \text{OSp}(N|4)_3, \text{SU}(2, 2|N)_4, \text{F}(4)_5, \text{OSp}(8|N)_6, \text{etc.}$
 $d > 6$ supergroups contain more than $\text{Spin}(d, 2) \rightarrow \Gamma^{M_1 \dots M_p} \text{D-branes}^4$

Therefore, in these cases the coupling $(L^{MN} + S^{MN}) \Gamma^{MN}$ shown in Eq. (5) takes the matrix form $2^{-[d/2]} (L^{MN} + S^{MN}) \begin{pmatrix} \Gamma^{MN} & 0 \\ 0 & 0 \end{pmatrix}$. This coupling scheme [24][2] applies to all the cases listed in the first column of the table Eq.(8) and to the $\text{SO}(11, 2)$ covariant $d = 11$ $\text{OSp}(1|64)$ toy M-model [20] in the second column.

For the $d = 10$ $\text{AdS}_5 \times \text{S}^5$ case listed in the second column the coupling scheme is slightly different. Namely, we do not keep full covariance $\text{SO}(10, 2)$ in 12 dimensions, but rather only under its subgroup $\text{SO}(4, 2) \times \text{SO}(6) = \text{SU}(2, 2) \times \text{SU}(4)$. The supergroup that contains this subgroup is $\text{PSU}(2, 2|4)$. To take this into account we split the 12 coordinates into two groups of six each $X^M = (X^m, X^I)$, and similarly for the momenta $P^M = (P^m, P^I)$, and associate the first six dimensions with the upper block of the $\text{PSU}(2, 2|4)$ matrix and the last six dimensions with the lower block (in place of the R-symmetry). Therefore, for $d = 10$ the coupling $(L^{MN} + S^{MN}) \Gamma^{MN}$ shown in Eq. (5) takes the matrix form $\frac{1}{4} (L^{mn} + S^{mn}) \begin{pmatrix} \Gamma^{mn} & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{4} (L^{IJ} + S^{IJ}) \begin{pmatrix} 0 & 0 \\ 0 & -\Gamma^{IJ} \end{pmatrix}$. This $d = 10$ 2T model, for $n = 0$ (i.e. no spin S^{MN} or S^{IJ}), produces the supersymmetric particle moving on $\text{AdS}_5 \times \text{S}^5$ in one of its holographic images, and its physical quantum spectrum is identical to the Kaluza-Klein towers of type IIB supergravity compactified on $\text{AdS}_5 \times \text{S}^5$ [10]. This is a model of interest in the context of the $d = 10$ twistor superstring described below, and the AdS-CFT correspondence. A similar model for the $d=11$ $\text{AdS}_4 \times \text{S}^7$ or $\text{AdS}_7 \times \text{S}^4$ listed in the table does not lead to the corresponding compactification of $d = 11$ supergravity, but rather to the first massive level of the $d=11$ supermembrane.

d=3:	$\text{Spin}(3, 2) = \text{Sp}(4) \subset \text{OSp}(N 4)$	d=10 : $\text{AdS}_5 \times \text{S}^5$	$\text{Spin}(4, 2) \times \text{Spin}(6) \subset \text{PSU}(2, 2 4)$
d=4:	$\text{Spin}(4, 2) = \text{SU}(2, 2) \subset \text{PSU}(2, 2 N)$	d=11 : $\text{AdS}_4 \times \text{S}^7$	$\text{Spin}(3, 2) \times \text{Spin}(8) \subset \text{OSp}(8 4)$
d=5	$\text{Spin}(5, 2) \subset \text{F}(4)$ it contains also $\text{SU}(2)$	d=11 : $\text{AdS}_7 \times \text{S}^4$	$\text{Spin}(6, 2) \times \text{Spin}(5) \subset \text{OSp}(8 4)$
d=6	$\text{Spin}(6, 2) \subset \text{OSp}(8 N)$	d=11:	toy M-model $\text{Spin}(11, 2) \subset \text{OSp}(1 64)$ with D-branes ⁴
any d	$\text{Spin}(d, 2) = G$ purely bosonic twistors in any d	etc.	generalizations of above

For $d > 6$ supergroups contain bosonic subgroups that are larger than $\text{SO}(d, 2)$ as long as we insist that $\text{SO}(d, 2)$ appears in the spinor representation⁴. The extra bosons contained in g correspond to D brane-like degrees of freedom. If we require full covariance for $\text{SO}(d, 2)$ we are forced to admit these as additional degrees of freedom beyond those of the superparticle. The toy M-model in $d = 11$ based on $\text{OSp}(1|64)$ is one of the most interesting cases of this type[20]. By breaking the covariance to a subgroup of $\text{SO}(d, 2)$ we can build 2T models such as the $\text{AdS}_{d-k} \times \text{S}^k$ cases based only on particle degrees of freedom similar to what was described above.

3.2 Twistor gauge

The Lagrangian in Eq. (5) has an evident global symmetry G_d which corresponds to group transformations on the right side of g . These leave the Cartan form $(\partial g) g^{-1}$ invariant. The conserved Noether charge for this symmetry is the supermatrix J_A^B in the Lie algebra of G_d

$$\text{Gauge invariant global symm} \quad J_A^B \sim \frac{i}{2} (L_{MN} + S_{MN}) (g^{-1} \Gamma^{MN} g)_A^B$$

The Lagrangian is also invariant under a number of local symmetries. To begin there is the built in $\text{OSp}(n|2)$ local supersymmetry on the worldline as in Eq. (6). In addition, there are local spacetime supersymmetries. Some of these become easier to notice by rewriting the Lagrangian (5) in the form

$$L = \frac{1}{2^{[d/2]}} q^{ab} \text{Tr} [\partial_\tau (g^{-1} Y_a \cdot \Gamma g) (g^{-1} Y_b \cdot \Gamma g)] - \frac{1}{2} A^{ab} Y_a \cdot Y_b$$

Then it is easy to see that there is an invariance under local transformations $[\text{Spin}(d, 2) \times \text{R-symmetry}] \in G_d$ that are simultaneously applied on the *left side* of g as well as

⁴Here we give a list of the smallest bosonic subgroups in a supergroup G that contain $\text{Spin}(d, 2)$ for $3 \leq d \leq 12$. We also list the generators, and their numbers in parentheses, as represented by antisymmetrized products of gamma matrices $\Gamma^{M_1 \dots M_n} \equiv \frac{1}{n!} \Gamma^{[M_1} \dots \Gamma^{M_n]}$ in dimension $d+2$ labelled by M

d	$\text{Spin}(d, 2)$	spinor	$\subseteq G_{\text{bose}}$	generators of G_{bose} in $\text{Spin}(d, 2)$ basis	contained in
3	$\text{Spin}(3, 2)$	4	$\text{Sp}(4, R)$	Γ^{MN} (10)	$(4 \times 4)_s$
4	$\text{Spin}(4, 2)$	4_\pm	$\text{SU}(2, 2)$	Γ^{MN} (15)	$(4 \times 4^*)_s$
5	$\text{Spin}(5, 2)$	8_+	$\text{SO}^*(8)$	Γ^{MN} (21) + Γ^M (7)	$(8 \times 8)_a$
6	$\text{Spin}(6, 2)$	8_+	$\text{SO}^*(8)$	Γ^{MN} (28)	$(8 \times 8)_a$
7	$\text{Spin}(7, 2)$	16	$\text{SO}^*(16)$	Γ^{MN} (36) + Γ^{MNK} (84)	$(16 \times 16)_a$
8	$\text{Spin}(8, 2)$	16_\pm	$\text{SU}^*(16)$	Γ^{MN} (45) + Γ^{MNKL} (210)	$(16 \times 16^*)_s$
9	$\text{Spin}(9, 2)$	32	$\text{Sp}^*(32)$	Γ^{MN} (55) + Γ^M (11) + $\Gamma^{M_1 \dots M_5}$ (462)	$(32 \times 32)_s$
10	$\text{Spin}(10, 2)$	32_+	$\text{Sp}^*(32)$	Γ^{MN} (66) + $\Gamma^{M_1 \dots M_6}$ (462)	$(32 \times 32)_s$
11	$\text{Spin}(11, 2)$	64	$\text{Sp}^*(64)$	Γ^{MN} (78) + Γ^{MNK} (286) + $\Gamma^{M_1 \dots M_6}$ (1716)	$(64 \times 64)_s$
12	$\text{Spin}(12, 2)$	64_\pm	$\text{SU}^*(64)$	Γ^{MN} (91) + Γ^{MNKL} (1001) + $\Gamma^{M_1 \dots M_6}$ (3003)	$(64 \times 64^*)_s$

The antisymmetric $\Gamma^{M_1 \dots M_n}$ are associated with group parameters $\omega_{M_1 \dots M_n}(\tau)$ that cannot be eliminated by the gauge symmetries, and therefore they are additional degrees of freedom analogous to D-brane collective coordinates.

on the M index of Y_a^M . There is also local fermionic kappa supersymmetry that is also applied on the left side of G_d as well as on A^{ab} as explained in [24][2][9]. Let us review what physical degrees of freedom remain after gauge degrees of freedom are removed. We consider $n = 0$ in what follows (i.e. supersymmetrizing the scalar particle), the general n is similar.

The local $\text{Spin}(d, 2) \times \text{R-symmetry}$ has enough gauge parameters to remove all of the bosons from g in all the cases listed in the table Eq.(8), except for the toy M-model which has D-branes⁴. Therefore for those cases the bosonic degrees of freedom are just the particle phase space (X^M, P^M) or their gauge equivalent.

The local kappa supersymmetry has enough fermionic gauge parameters to remove 3/4 of the fermions Θ shown in Eq.(7) for $d = 3, 4, 5, 6$ and the $d = 11$ toy M-model. On the other hand for the $d = 10$ $\text{AdS}_5 \times \text{S}^5$ case only 1/2 of the 32 fermions can be removed by the kappa supersymmetry, while for the $d = 11$ $\text{AdS}_4 \times \text{S}^7$ or $\text{AdS}_7 \times \text{S}^4$ there is no kappa supersymmetry at all.

This summarizes then the physical degrees of freedom up to gauge equivalences. The interesting aspect of 2T-physics is that the gauge equivalence within the 2T-system does not necessarily imply that the 1T interpretation is the same, but rather that there are dualities between various holographic 1T images, as in the figure above.

It was shown in [24][2][9] that, for $n = 0$ and $d = 3, 4, 5, 6$, one can choose a gauge that reduces the 2T system to the standard superparticle in the corresponding number of dimensions. Also it was shown in [10] that the $d = 10$ $\text{SU}(2, 2|4)$ case can be gauge fixed to the superparticle moving on the $\text{AdS}_5 \times \text{S}^5$ background.

In this lecture we will concentrate on the twistor gauge [2][9] for the $n = 0$ case. By using the local symmetry $\text{Sp}(2, R) \times \text{Spin}(d, 2)$, and the constraints $X \cdot P = X^2 = P^2 = 0$, we can gauge fix $X^M(\tau)$, $P^M(\tau)$ for all τ to the following trivial configuration

$$X^M = (X^{+'}, \overset{-}{0}, \overset{-}{0}, \overset{-}{0}, \overset{-}{0}), \quad P^M = (\overset{+}{0}, \overset{-}{0}, P^+, \overset{-}{0}, \overset{-}{0}), \quad i = 1, \dots, (d-2). \quad (9)$$

In this gauge the purely bosonic system for any d , the supersymmetric systems for $d = 3, 4, 6$, and the $d = 11$ toy M-model listed in Eq. (8) collapse to the form⁵

$$L = -\frac{L^{+'+}}{2^{[d/2]-1}} \text{Tr} \left(\partial_\tau g g^{-1} \begin{pmatrix} \Gamma^{-'-} & 0 \\ 0 & 0 \end{pmatrix} \right) = -i \partial_\tau \bar{Z}^{aA} Z_{Aa}, \quad a = 1, \dots, k \quad (10)$$

$$J_A^B \sim L^{+'+} \left(g^{-1} \begin{pmatrix} \Gamma^{-'-} & 0 \\ 0 & 0 \end{pmatrix} g \right)_A^B = -2 Z_{Aa} \bar{Z}^{aB}, \quad \bar{Z}^{aA} Z_{Ab} = 0 \quad (11)$$

In an appropriate basis for gamma matrices⁶ we find $\Gamma^{-'-} = 2\tau^- \otimes \sigma^- \otimes 1_k$ which is

⁵For the $d = 10$ $\text{AdS}_5 \times \text{S}^5$ case the gauge fixed forms of X, P are different as given in Eq.(17).

⁶ $d+2$ gamma matrices that satisfy $\Gamma^M \bar{\Gamma}^N + \Gamma^N \bar{\Gamma}^M = 2\eta^{MN}$ are chosen in a Weyl basis as follows

$$\Gamma^{\pm'} = (\pm\sqrt{2}\tau^{\pm}) \otimes 1 \otimes 1_k, \quad \Gamma^{\pm} = \tau_3 \otimes (\pm\sqrt{2}\sigma^{\pm}) \otimes 1_k, \quad \Gamma_i = \tau_3 \otimes \sigma_3 \otimes \gamma_i, \quad i = 1, \dots, (d-2)$$

γ_i is $k \times k$ matrix

In odd dimensions we have $\bar{\Gamma}^M = \Gamma^M$. In even dimensions $\bar{\Gamma}^M$ differs from Γ^M only for the last gamma matrix, which is proportional to the identity $1_{4k} = 1_2 \otimes 1_2 \otimes 1_k$, namely $\Gamma_{d-2} = i1_{4k} = -\bar{\Gamma}_{d-2}$. For example for $d = 4$ or $\text{SO}(4, 2) = \text{SU}(2, 2)$, we have $k = 1$, we choose $\gamma_1 = -1$, and take $\Gamma_2 = i1_4 = -\bar{\Gamma}_2$. Then we construct $\Gamma^{MN} = \frac{1}{2} (\Gamma^M \bar{\Gamma}^N - \Gamma^N \bar{\Gamma}^M)$ as 4×4 matrices.

a $4k \times 4k$ matrix with lots of zeroes and k nonzeros off the diagonal. Therefore only certain off-diagonal rows of g denoted by \bar{Z}^{aA} and certain off-diagonal columns of g^{-1} denoted by Z_{Aa} contribute in the trace in L or to J_A^B . Also the relation $gg^{-1} = 1$ implies the constraint $\bar{Z}^{aA}Z_{Ab} = 0$ as an off diagonal entry in the matrix 1. The A, B indices label the fundamental representation of G_d and therefore the Z_{Aa} denote k supertwistors with $a = 1, \dots, k$. Thus the theory has now been written in terms of twistors.

Note that for $d = 4$ the group is $\text{PSU}(2, 2|4)$, the gamma matrices are 4×4 , and $k = 1$. Therefore for $d = 4$ there is a single supertwistor Z_A in the fundamental representation of $\text{PSU}(2, 2|4)$ and it is constrained by $\bar{Z}^A Z_A = 0$. These constrained twistors describe $\text{CP}^{3|4}$. Thus the 2T formalism for supertwistors is in full agreement with the expectation about twistors in four dimensions. The 2T formalism gives the appropriate generalization to all the other dimensions mentioned earlier. These will be described below case by case for a few dimensions of special interest.

The Lagrangian in (10) suggests that \bar{Z}^{aA} is the canonical conjugate to Z_{Aa} and therefore the twistors can be expressed in terms of oscillators. The current J_A^B for the global symmetry in Eq.(11) is constructed from these oscillators, and the quantum states are obtained in the Fock space of these oscillators. The physical states are the subset of the Fock space that satisfies the constraint $\bar{Z}^{aA}Z_{Ab} = 0$, and form a unitary representation of the global symmetry G_d . This setup precisely coincides with the Bars-Günaydin (BG) oscillator approach to unitary representations of supergroups developed in 1983 [12]. The additional constraint is a gauge invariance condition and is implemented by following the discussion about “color” in the improved oscillator formalism given in [11]. Therefore, we can easily obtain the quantum spectrum and compare to the quantum spectrum in another gauge, such as the superparticle gauge. The agreement is perfect as expected from the 2T approach, since each gauge corresponds to a holographic image of the same 2T system.

The supertwistors are constrained as shown above. The full solution of these constraints in terms of unconstrained degrees of freedom is given as a coset $T_\Gamma \in G_d/H_\Gamma$, where H_Γ is a gauged subgroup H_Γ of G_d , that is a remainder of the original gauge symmetries mentioned before. H_Γ is identified as the subgroup that contains all the generators of G_d that commute with the generator represented by $\begin{pmatrix} \Gamma'^{-} & 0 \\ 0 & 0 \end{pmatrix}$. We can then show [9][3] that the Lie algebras of h_Γ and of the coset t_Γ form triangular sub-supergroups, and they satisfy (anti)commutation rules of the type [9][3]

$$[h_\Gamma, h_\Gamma] \sim h_\Gamma, \quad [t_\Gamma, t_\Gamma] \sim t_\Gamma, \quad [h_\Gamma, t_\Gamma] \sim h_\Gamma + t_\Gamma \quad (12)$$

Furthermore the system can be written in terms of the unconstrained degrees of freedom $t \in G_d/H_\Gamma$ in the form

$$L = -\frac{L^{++}}{2^{[d/2]-1}} \text{Tr} \left(\partial_\tau t t^{-1} \begin{pmatrix} \Gamma'^{-} & 0 \\ 0 & 0 \end{pmatrix} \right), \quad J_A^B \sim L^{++} \left(t^{-1} \begin{pmatrix} \Gamma'^{-} & 0 \\ 0 & 0 \end{pmatrix} t \right)_A^B \quad (13)$$

This is like a sigma model based on a coset but the Lagrangian is linear in the Cartan connection $\partial_\tau t t^{-1}$ (as opposed to quadratic form for the sigma model) and there is an unusual insertion $\begin{pmatrix} \Gamma'^{-} & 0 \\ 0 & 0 \end{pmatrix}$.

3.3 Supertwistors for d=4 and SYM spectrum

The twistor must reproduce the physical degrees of freedom and quantum states of the corresponding $d = 4$, $N = 4$ superparticle, as expected from the 2T formalism. Let's see how this is obtained explicitly.

To begin the superparticle has $4x, 4p$ and 16θ real degrees of freedom in super phase space. We remove $1x$ and $1p$, due to τ reparametrization and the corresponding $p^2 = 0$ constraint. We also remove 8 fermionic degrees of freedom due to kappa supersymmetry. We are left over with $3x, 3p, 8\theta$ physical degrees of freedom. With these we construct the physical quantum states as an arbitrary linear combination of the basis states in momentum space $|\vec{p}, \alpha\rangle$, where α is the basis for the Clifford algebra satisfied by the 8θ . This basis has 8 bosonic states and 8 fermionic states. Viewed as probability amplitudes in position space $\langle x, \alpha | \psi \rangle$ these are equivalent to fields $\psi(x)_{8_B+8_F}$ which correspond to the independent *solutions* of all the constraints. One finds that these are the same as the 8 bose and 8 fermi fields of the Super Yang Mills (SYM) theory which are the solutions of the linearized equations of motion in the lightcone gauge. They consist of two helicities of the gauge field $A_{\pm 1}(x)$, two helicities for the gauginos $\psi_{+\frac{1}{2}}^a(x)$, $\bar{\psi}_{-\frac{1}{2},a}(x)$ in the $\mathbf{4}, \bar{\mathbf{4}}$ of $SU(4)$, and six scalars $\phi^{[ab]}(x)$ in the $\mathbf{6}$ of $SU(4)$.

Now we count the physical degrees of freedom for the twistors. We have already explained following Eq. (10) that for $d = 4$ we have one complex twistor Z_A in the fundamental representation of $PSU(2, 2|4)$, with a Lagrangian and a conserved current given by

$$L = i\bar{Z}^A \partial_\tau Z_A, \quad J_A^B = -2Z_A \bar{Z}^B, \quad \text{and} \quad \bar{Z}^A Z_A = 0 \quad (14)$$

Z_A is in fundamental representation of $PSU(2, 2|N) \leftrightarrow CP^{3|N}$

To start Z_A has 4 complex bosons and 4 complex fermions, i.e. $8_B + 8_F$ real degrees of freedom. However, there is one constraint $\bar{Z}^A Z_A = 0$ and a corresponding $U(1)$ gauge symmetry⁷, which remove 2 bosonic degrees of freedom. The result is $6_B + 8_F$ physical degrees of freedom which is equivalent to $CP^{3|4}$, and the same number as $3x, 3p, 8\theta$ for the superparticle, as expected. Instead of constrained twistors we can also express the $CP^{3|4}$ theory in terms of unconstrained coset parameters in the form (12,13) where h_Γ was given in [9], and with more details in [3].

To construct the spectrum we could resort to well known twistor techniques by working with fields $\phi(Z)$ that are holomorphic in Z_A on which \bar{Z}^A acts as a derivative $\bar{Z}^A \phi(Z) = \partial \phi(Z) / \partial Z_A$, as dictated by the canonical structure that follows from the Lagrangian (14). Imposing the constraint amounts to requiring $\phi(Z)$ to be homogeneous with a given degree h , namely $Z_A \bar{Z}^A \phi(Z) = Z_A \partial \phi(Z) / \partial Z_A = h \phi(Z)$. Only one value of h is permitted. Naively h is zero at the classical level, but at the quantum level we have to determine the correct value of h that may arise due to quantum ordering. In the case of the $d = 4$ $N = 4$ superparticle described by the $PSU(2, 2|4)$ twistor indeed we find $h = 0$, and the resulting spectrum is again the

⁷This can be restated by reformulating the above system by rewriting $L = \bar{Z}^A (\partial_\tau + A) Z_A$ with an extra $U(1)$ gauge field A , and deriving the constraint by varying A .

SYM fields. This is the degree zero wavefunction $\phi(Z)$ described in [8]. Recall that in [8] there are also twistor wavefunctions $f(Z), g(Z)$ that describe the spectrum of conformal gravity; those arise in the same twistor formalism, but at a different value of h . However, since only one value of h is permitted in the current superparticle model, only SYM states $\phi(Z)$ are present. Of course, this is no surprise in the 2T setting. We have simply compared two gauges, and we must agree.

It is also worth analyzing the quantum system in terms of oscillators related to twistors and understand the unitarity of the physical space. We emphasize that \bar{Z}^A is obtained from Z_A by hermitian conjugation and multiplying by the $\text{PSU}(2, 2|4)$ metric. To see the oscillator formalism clearly we work in a basis of $\text{SU}(2, 2|4)$ in which the metric is $\text{diag}(1_2, -1_2, 1_4)$. This is the $\text{SU}(2) \times \text{SU}(2)$ basis for $\text{SU}(2, 2)$, to be contrasted with the $\text{SL}(2, C)$ basis in which the metric is off-diagonal and usually used to discuss Lorentz covariant twistors. The two bases are simply related by a linear transformation that diagonalizes the metric. In this diagonal basis we identify the oscillators as (a bar over the symbol means hermitian conjugation)

$$L = i\bar{Z}^A \partial_\tau Z_A = i\bar{a}^i \partial_\tau a_i - ib_I \partial_\tau \bar{b}^I + i\bar{\psi}^r \partial_\tau \psi_r \quad \begin{array}{l} \text{SU}(2) \\ i = 1, 2, \end{array} \quad \begin{array}{l} \text{SU}(2) \\ I = 1, 2 \end{array} \\ r = 1, \dots, 4 \text{ SU}(4)$$

$$Z_A = \begin{pmatrix} a_i \\ \bar{b}^I \\ \psi_r \end{pmatrix}, \quad \bar{Z}^A = (\bar{a}^j, -b_J, \bar{\psi}^s), \quad J_A^B = -2Z_A \bar{Z}^A = -2 \begin{pmatrix} a_i \bar{a}^j & -a_i b_J & a_i \bar{\psi}^s \\ \bar{b}^I \bar{a}^j & -\bar{b}^I b_J & \bar{b}^I \bar{\psi}^s \\ \psi_r \bar{a}^j & -\psi_r b_J & \psi_r \bar{\psi}^s \end{pmatrix}$$

It is significant to note that, after taking care of the metric in \bar{Z} as above, the usual canonical rules applied to this Lagrangian identifies the oscillators as being all *positive norm* oscillators $[a_i, \bar{a}^j] = \delta_i^j$, $[b_I, \bar{b}^J] = \delta_I^J$ and $\{\psi_r, \bar{\psi}^s\} = \delta_r^s$. Therefore all Fock states have positive norm. However, among them we must choose only those that satisfy the constraints

$$0 = \bar{Z}^A Z_A = \bar{a}^i a_i - b_I \bar{b}^I + \bar{\psi}^r \psi_r = \bar{a}^i a_i - (\bar{b}^I b_I + 2) + \bar{\psi}^r \psi_r \\ \Leftrightarrow 2 = N_a + N_\psi - N_b \equiv \Delta, \quad N_a, N_\psi, N_b \text{ number operators}$$

This is precisely the BG oscillator formalism for unitary representations of noncompact groups [12] for a single “color”. The constraint $\Delta = 2$ is discussed in [11]. These physical states are characterised by identifying the following lowest supermultiplet

$$\Delta = 2 : \left(\begin{array}{ccccc} A_{+1} & \psi_{+1/2}^r & \phi^{[rs]} & \psi_{-1/2, a} & A_{-1} \\ \bar{a}^i \bar{a}^j & \bar{a}^i \bar{\psi}^r & \bar{\psi}^r \bar{\psi}^s & \bar{b}^I \bar{\psi}^r \bar{\psi}^s \bar{\psi}^m & \bar{b}^I \bar{b}^J \bar{\psi}^r \bar{\psi}^s \bar{\psi}^m \bar{\psi}^n \\ (1, 0, 1) & (\frac{1}{2}, 0, 4) & (0, 0, 6) & (0, \frac{1}{2}, \bar{4}) & (0, 1, 1) \end{array} \right) |0\rangle$$

which is annihilated by the double annihilation generators $a_i b_J$ which is part of J_A^B in the conformal subgroup $\text{SU}(2, 2)$. All other states with $\Delta = 2$ are descendants obtained by applying arbitrary powers of the double creation generator $\bar{a}^j \bar{b}^I$ in $\text{SU}(2, 2)$. The lowest states correspond to the SYM fields, the descendants are analogous to applying multiple derivatives on a field. The classification of the lowest states under $\text{SU}(2) \times \text{SU}(2) \times \text{SU}(4) \subset \text{PSU}(2, 2|4)$ is given under each combination of oscillators in the form $(j_1, j_2, \dim(\text{SU}(4)))$ where j_1, j_2 correspond to the spin quantum numbers

in each $SU(2)$. In arriving at these quantum numbers we took into account that $\bar{a}^i \bar{a}^j$ is symmetric while $\bar{\psi}^r \bar{\psi}^s$ is antisymmetric, etc. Above each of the oscillator combination we indicated one of the physical helicity components of the SYM fields associated with that state. This is because in comparing the compact $SU(2) \times SU(2) \subset SU(2, 2)$ to the *helicity* embedded in the noncompact Lorentz group $SL(2, C) \subset SU(2, 2)$ we must identify as helicity only the spin up part from the first $SU(2)$ and the spin down part from the second $SU(2)$.

Although we gave a whole supermultiplet of lowest states above, there really is only one lowest oscillator state for the entire unitary representation of $PSU(2, 2|4)$. That one is simply $\bar{\psi}^r \bar{\psi}^s |0\rangle$, which satisfies $\Delta = 2$. All other states with $\Delta = 2$ are obtained by applying all powers of J_A^B on this state (note $[\Delta, J_A^B] = 0$). This is called the doubleton representation of $PSU(2, 2|4)$ (sometimes also called the singleton). If we watch carefully the orders of the oscillators we can show that the generators of $PSU(2, 2|4)$ in this representation satisfy [11] the following nonlinear constraints

$$(JJ)_A^B = 4(J)_A^B + 0 \quad (15)$$

The linear J follows from the commutation rules among the generators, the coefficient 4 is related to overall normalization of J , while the coefficient 0 is the $PSU(2, 2|4)$ quadratic Casimir eigenvalue $C_2 = 0$. This equation should be viewed as a set of constraints that are satisfied by the generators in this particular representation, and as such this relation identifies uniquely the representation (there is a unique $C_2 = 0$ representation if we also specify the $SU(2, 2)$ conformal dimension=1). If the theory is expressed in any other form (such as particle description, or field theory) the doubleton representation can be identified in terms of the global symmetry as one that must satisfy the constraints (15), automatically requiring the 6 scalars $\phi^{[ab]}$ as the lowest $SU(4)$ multiplet. This is a completely $PSU(2, 2|4)$ covariant and gauge invariant way of identifying the physical and unitary states of the theory. Of course the $d = 4$, $N = 4$ SYM fields satisfy this criterion.

3.4 Supertwistors for d=6 and self dual supermultiplet

Now that the concepts have been illustrated clearly for $d = 4$, we summarize quickly the equivalent statements for $d = 6$. The superparticle in $d = 6$ and $N = 4$ starts out with $6x, 6p, 16\theta$ real degrees of freedom. Fixing τ , and kappa local gauges and solving constraints, reduces the physical degrees of freedom down to $5x, 5p, 8\theta$. The superparticle action has a hidden global superconformal symmetry $OSp(8^*|4)$ [24][2], therefore the physical states should be classified as a unitary representation under this group.

If we quantize in the lightcone gauge we find $8_B + 8_F$ states which should be compared to the fields of a six dimensional field theory taken in the lightcone gauge. There are two possible candidates; (1) the linearized six dimensional SYM theory with $N = 4$ SUSY in the lightcone gauge, and (2) the self dual supermultiplet classified (covariantly) as

$$\begin{aligned} &SO(5,1) \times Sp(4): F_{[\mu\nu\lambda]}^+, \psi_\alpha^a, \phi^{[ab]} \\ &\text{self dual } F_{[\mu\nu\lambda]}^+ = \partial_{[\mu_1} A_{\mu_2\mu_3]} = \varepsilon_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6} \partial^{[\mu_4} A^{\mu_5\mu_6]} \end{aligned} \quad (16)$$

The SYM lightcone degrees of freedom consists of $8_B + 8_F$, with the 8 bosons being: the transverse 4-vector A_i in $SO(4) \subset SO(5,1)$ and four real scalars ϕ^I in an internal compact $SO(4)$. On the other hand for the self dual multiplet, we have the following lightcone fields: a self dual antisymmetric tensor $A_{ij} = i\varepsilon_{ijkl}A^{kl}$ in $SO(4) \subset SO(5,1)$ (i.e. 3 fields), and the $Sp(4)$ traceless antisymmetric $\phi^{[ab]}$ (5 scalars). These are clearly different. Only the self dual supermultiplet is consistent with the compact $USp(4) \subset OSp(8^*|4)$ classification (the fundamental **4** of $USp(4)$ is not real but pseudo-real, while the **5** represented as $\phi^{[ab]}$ is real). Therefore the initial hidden superconformal symmetry $OSp(8^*|4)$ of the superparticle (and of the 2T superparticle) is consistent only with the field theory for the $d = 6$ self dual supermultiplet. An *interacting* quantum conformal field theory with these degrees of freedom is expected but cannot be written down covariantly in the form of a field theory [14].

Let us now examine the twistors that emerged in Eq. (10) for this case. We have

$Z_{Aa} = \begin{pmatrix} 8_{bose} \\ 4_{fermi} \end{pmatrix}$	12x2 rectangular matrix, $A=1,\dots,12$; $a=1,2$ 2 twistors in fundamental rep of $OSp(8^* 4)$
$Z_{Aa} = (12,2)$	of $OSp(8^* 4)_{\text{global}} \times SU(2)_{\text{local}}$
$L = \bar{Z}^{Aa} ((\partial + A) Z)_{Aa} \rightarrow \bar{Z}^{aA} Z_{Ab} = 0,$	$SU(2)$ gauge invariants in Fock space
Pseudo-real 1st & 2nd related	$Z_{Aa} = \begin{pmatrix} a_{1i} & a_{2i} \\ \bar{a}_2^i & -\bar{a}_1^i \\ \psi_{1\alpha} & \psi_{2\alpha} \\ \bar{\psi}_2^\alpha & -\bar{\psi}_1^\alpha \end{pmatrix}$ <div style="display: inline-block; vertical-align: middle; margin-left: 10px;"> $i: 4 \text{ of } SU(4) \subset SO(6,2)$ $\alpha: 2 \text{ of } SU(2) \subset Sp(4)$ </div>

Here \bar{Z}^{Aa} is obtained from Z_{Aa} by taking hermitian conjugation and multiplying by the 12×12 diagonal matrix $diag(1_4, -1_4, 1_2, 1_2)$. However, Z_{Aa} is pseudo real, $\bar{Z}^{aA} = C^{AB} Z_{Bb} \varepsilon^{ba}$, as it is defined as part of the group element $g \in OSp(8^*|4)$ with the correct signature. Then Z_{Aa} takes the form above in a natural basis. Thus the second column is related to the first one, but still consistent with a local $SU(2)$ applied on $a = 1, 2$. When Z, \bar{Z} of these forms are inserted in the Lagrangian, it is seen that according to the canonical formalism, the oscillators identified above all have positive norm $[a_{1i}, a_1^j] = \delta_i^j = [a_{2i}, a_2^j]$, $\{\psi_{1\alpha}, \bar{\psi}_1^\beta\} = \delta_\alpha^\beta = \{\psi_{2\alpha}, \bar{\psi}_2^\beta\}$. We count the degrees of freedom before the constraints, and find that Z_{Aa} has $(8_B + 4_F) \times 2_{(\text{complex})} = 16_B + 8_F$ (namely $a_{1i}, a_{2i}, \psi_{1\alpha}, \psi_{2\alpha}$). The constraints are due to a $SU(2)$ gauge symmetry acting on the index $a = 1, 2$ (although it seems like $SU(2) \times U(1)$, the $U(1)$ part is automatically satisfied because of the pseudoreal form of Z_{Aa}). The 3 gauge parameters and 3 constraints remove 6 bosonic degrees of freedom, and we remain with $10_B + 8_F$ physical degrees of freedom. This is the same as the count for the superparticle $(5x, 5p, 8\theta)$. It is obvious we have the same number of degrees of freedom and the same symmetries $OSp(8^*|4)$, with the symmetry being much more transparent in the twistor basis.

Instead of constrained twistors we can also express this theory in terms of unconstrained coset parameters in the form (12,13) where h_Γ was given in [9], with more details in [3].

The quantum theory can proceed in terms of twistors or in terms of constrained oscillators. The resulting representation, after satisfying the $SU(2)$ constraints in the

Fock space of the oscillators defined above, is precisely the doubleton of $\text{OSp}(8^*|4)$, and this is precisely equivalent to the fields in Eq. (16). This oscillator representation was worked out a long time ago in [13] using again the BG method [12]. The selection of the doubleton among many other Fock space states discussed in [13] is the analog of choosing the $\text{SU}(2)$ “color” singlet in analogous discussion to the one in [11]. More details will be given in [3].

3.5 Supertwistors for $d=10$ and $\text{AdS}_5 \times \text{S}^5$ KK towers

This was explained in detail in [10]. Here we will quickly count degrees of freedom for the $\text{AdS}_5 \times \text{S}^5$ superparticle. This superparticle starts out with $10x, 10p, 16\theta$ real degrees of freedom. Fixing τ gauges and solving constraints, reduces the physical degrees of freedom down to $9x, 9p, 16\theta$. With 16θ 's we construct the Clifford algebra that is realized on states with $128_B + 128_F$. These correspond to the supergravity multiplet. Hence this case is related to gravity.

The superparticle action has a hidden global superconformal symmetry $\text{PSU}(2, 2|4)$ whose generators are given in [10], therefore the physical states should be classified as a unitary representation under this group. The resulting spectrum coincides with all the infinite Kaluza-Klein towers of type IIB supergravity compactified on $\text{AdS}_5 \times \text{S}^5$.

Now we turn to the twistor gauge. This is a different gauge choice compared to (9). We have split phase space into two groups of six each, as $X^M = (X^m, X^I)$, $P^M = (P^m, P^I)$, but the $\text{Sp}(2)$ constraints is $\text{SO}(10, 2)$ covariant for the overall 12 dimensions $X^2 = P^2 = X \cdot P$. Using the $\text{Sp}(2) \times \text{SO}(4, 2) \times \text{SO}(6)$ gauge symmetries we choose gauges and solve all the $\text{Sp}(2)$ constraints with the following configuration

$$\begin{aligned} M &= (0' \ 0 \ 1 \ \cdots \ 4 \ , \quad 5 \ 6 \ 7 \ \cdots \ 10) \\ X^M &\sim (a \ 0 \ 0 \ \cdots \ 0 \ , \quad a \ 0 \ 0 \ \cdots \ 0) \\ P^M &\sim (0 \ b \ 0 \ \cdots \ 0 \ , \quad 0 \ b \ 0 \ \cdots \ 0) \end{aligned} \quad (17)$$

In this configuration the only nonzero angular momenta that couple according to the scheme given above $L^{0'0} = ab \equiv l$ and $L^{56} = ab \equiv l$. Therefore, instead of Eqs. (11,10) we now obtain, with $g \in \text{PSU}(2, 2|4)$,

$$\begin{aligned} L &= -\frac{l}{2} \text{Str} \left(\partial_\tau g g^{-1} \hat{\Gamma} \right), \quad J_A^B \sim \left(g^{-1} \hat{\Gamma} g \right)_A^B, \quad \hat{\Gamma} \equiv \begin{pmatrix} \Gamma^{0'0} & 0 \\ 0 & -\Gamma^{56} \end{pmatrix}, \\ g &\in \text{PSU}(2, 2|4) / [\text{PSU}(2|2) \times \text{PSU}(2|2)] \end{aligned} \quad (18)$$

In an appropriate basis $\Gamma^{0'0}, \Gamma^{56}$ can be taken to be diagonal 4×4 matrices, each with two $+1$ and two -1 eigenvalues, therefore $\hat{\Gamma}$ is the diagonal matrix $\text{diag}(1_2, -1_2, -1_2, 1_2)$. Any generator that commutes with this matrix is a remaining gauge symmetry. Thus, there is still the gauge symmetry $[\text{PSU}(2|2) \times \text{PSU}(2|2)]$. The first $\text{PSU}(2|2)$ acts on rows 1, 2, 7, 8 of g ($+1$ eigenvalues of $\hat{\Gamma}$) while the second $\text{PSU}(2|2)$ acts on rows 3, 4, 5, 6 (-1 eigenvalues of $\hat{\Gamma}$). After removing the gauge degrees of freedom the g in Eq.(18) belongs only to the coset $g \in \text{PSU}(2, 2|4) / [\text{PSU}(2|2) \times \text{PSU}(2|2)]$. We count the number of physical degrees of freedom as follows. The full $\text{PSU}(2, 2|4)$ supergroup

contains $15 + 15$ real bosons and 16 complex fermions, thus altogether $30_B + 32_F$. The gauge subgroup $PSU(2|2)$ contains $3+3$ bosons and 4 complex fermions. Therefore $PSU(2|2) \times PSU(2|2)$ has $12_B + 16_F$ real gauge degrees of freedom. The physical degrees of freedom in the coset is the difference, namely $18_B + 16_F$. As expected this is the same number as the $9x, 9p, 16\theta$ we counted for the $AdS_5 \times S^5$ superparticle above. The hidden $SU(2, 2|4)$ symmetries of the superparticle are evident in this twistor gauge. Hence, one alternative description of $AdS_5 \times S^5$ is the coset given above. This is a new observation.

We can now rewrite this in terms of constrained twistors, as we did for $d = 4, 6$ above. Rather than removing all of the gauge degrees of freedom we allow some of it to remain. Then from g we can extract four twistors which we call Z_{Aa} with the following properties⁸

$Z_{Aa} = \begin{pmatrix} \overset{a=1,2}{\text{bose}} & \overset{a=3,4}{\text{fermi}} \\ \text{fermi} & \text{bose} \end{pmatrix}$	$\begin{matrix} 8 \times 4 \text{ rectangular matrix} \\ 4 \text{ fundamental reps of } PSU(2,2 4) \end{matrix}$	(19)
$Z_{Aa} = (8,4) \text{ of } PSU(2,2 4)_{global} \times [PSU(2 2) \times U(1)]_{local}$		
$L = \bar{Z}^{Aa} ((\partial + A) Z)_{Aa} \rightarrow \bar{Z}^{aA} Z_{Ab} = 0, \quad \begin{matrix} \text{take gauge invariants} \\ \text{in Fock space} \end{matrix}$		

The first two twistors $a = 1, 2$ each has four bosons in 4 of $SU(2, 2)$ and four fermions in 4 of $SU(4)$. The last two twistors $a = 3, 4$ each has four fermions in 4 of $SU(2, 2)$ and four bosons in 4 of $SU(4)$. On the basis $a = 1, 2, 3, 4$ we act with a gauge symmetry $PSU(2|2) \times U(1)$, hence the Lagrangian is invariant under $PSU(2, 2|4)_{global} \times [PSU(2|2) \times U(1)]_{local}$. Let us count the degrees of freedom. The complex Z_{Aa} has $(16_B + 16_F) \times 2_{(\text{complex})}$, therefore $32_B + 32_F$ real degrees of freedom. $PSU(2|2) \times U(1)$ has $(3+3+1)_B + 8_F$ real gauge parameters. The gauge parameters together with the corresponding constraints remove $14_B + 16_F$. Therefore the physical degrees of freedom in Z_{Aa} is the difference, namely $18_B + 16_F$, which is the same as the $9x, 9p, 16\theta$ we counted for the $AdS_5 \times S^5$ superparticle above. The global symmetry is still $PSU(2, 2|4)$, and it has become evident rather than hidden in the twistor version we have just described. Hence, a new alternative description of $AdS_5 \times S^5$ is the constrained twistors Z_{Aa} given above.

The quantum theory for this case can again be described by using the BG oscillator approach [12][11]. But now the “color” group is the supergroup $PSU(2|2) \times U(1)$, and the discussion in [11] should be modified accordingly. The “color” supergroup is mathematically a new case in the BG approach, and will be further analyzed in [3].

3.6 Bosonic twistors in any d

The methods above can be applied in any dimension with the purely bosonic group $G = SO(d, 2)$ as listed in Eq.(8). In this case the analogs of the twistors Z_{Aa} correspond again to $1/4$ of the columns of g^{-1} . But for sufficiently large d (namely $d > 6$) there are more entries Z_{Aa} than there are group parameters in $SO(d, 2)$; hence these Z_{Aa} come out with lots of constraints among the entries in these rectangular matrices, as dictated by the spinor representation of the group $SO(d, 2)$. The independent

⁸These correspond to the four middle columns of g^{-1} , or equivalently the first two and last two columns of g^{-1} , as discussed in [9].

parameters correspond to the coset t_r as in Eqs. (12,13). These give the correct generalizations of twistors in the sense that they provide an alternative (twistor) description of the massless relativistic particle in d dimensions. It is a holographic image in the 2T structure. As d gets larger and larger beyond $d = 6$ it becomes cumbersome to try to describe these in terms of oscillators, because of the complexity of the constraints on the oscillators. However, if we relax the requirement of only particles, and admit also D-brane degrees of freedom, as in footnote (4), then the oscillator (or twistor) approach becomes again a very efficient tool. The oscillator version including D-branes was described in [2] for the group $\text{OSp}(M|2N)$, and applied to the toy M-model for $\text{OSp}(1|64)$ in [20]. A more detailed discussion of the twistors for general d , without and with D -branes, will appear in [3].

4 2T superstrings d=3,4,5,6,10

So far in this lecture we discussed superparticles and the associated supertwistors, and their physical spectra. These have a direct generalization to superstrings via the 2T superstring formalism given in [9]. Briefly, the action is

$$L_{2T}^{\pm} = \partial_{\pm} X \cdot P^{\pm} - \frac{1}{2} A X \cdot X - \frac{1}{2} B_{\pm\pm} P^{\pm} \cdot P^{\pm} - C_{\pm} P^{\pm} \cdot X \\ - \frac{1}{2[d/2]-1} \text{Str} \left(\partial_{\pm} g \bar{g} \begin{pmatrix} L_{MN}^{\pm} \Gamma^{MN} & 0 \\ 0 & 0 \end{pmatrix} \right) + L_G$$

$X_M(\tau, \sigma)$, $P_M^{\pm}(\tau, \sigma)$, $L_{MN}^{\pm} = X_{[M} P_{N]}^{\pm}$, $g(\tau, \sigma)$ are now string fields, and $\partial_{\pm} = \frac{1}{2}(\partial_{\sigma} \pm \partial_{\tau})$. Here L_{2T}^{\pm} represent left/right movers, and similar to [6] there is open string boundary conditions. L_G is an additional degree of freedom that describes an internal current algebra for some SYM group G . The local and global symmetries are similar to those of the particle and are described in [9]. The global symmetry is G_d chosen for various d as before, $G_d = \text{OSp}(8|4)_3, \text{SU}(2,2|4)_4, \text{F}(4)_5, \text{OSp}(8|4)_6$. The particle gauge for these give usual type superstrings and the twistor gauge gives twistor superstrings, with the twistors described above. In the 2T philosophy each one of these have many duals that can be found and investigated by choosing various gauges. This is a completely open field of investigation at this time.

Similarly to the $d = 10$ particle case we also consider the d=10 2T superstring

$$L_{2T}^{\pm} = \partial_{\pm} \hat{X} \cdot \hat{P}^{\pm} - \frac{1}{2} A \hat{X} \cdot \hat{X} - \frac{1}{2} B_{\pm\pm} \hat{P}^{\pm} \cdot \hat{P}^{\pm} - C_{\pm} \hat{P}^{\pm} \cdot \hat{X} \\ - \frac{1}{8} \text{Str} \left(\partial_{\pm} g \bar{g} \begin{pmatrix} L_{MN}^{\pm} \Gamma^{MN} & 0 \\ 0 & -L_{IJ}^{\pm} \Gamma^{IJ} \end{pmatrix} \right)$$

where $\text{SO}(10,2) \rightarrow \text{SO}(4,2) \times \text{SO}(6)$, $\hat{X}^M = (X^m, X^I)$, $\hat{P}_M^{\pm} = (P_m^{\pm}, P_I^{\pm})$, $g(\tau, \sigma) \in \text{SU}(2,2|4)$, and $L_{MN}^{\pm} = X_{[M} P_{N]}^{\pm}$, $L_{IJ}^{\pm} = X_{[I} P_{J]}^{\pm}$. The local and global symmetries are discussed in detail in [10],[9]. In the particle-type gauge, the spectrum in the particle limit is the same as linearized type IIB SUGRA compactified on $\text{AdS}_5 \times \text{S}^5$. In the twistor gauge this theory is currently being investigated by using the twistors in Eq. (19). As usual, being a 2T theory we expect dual versions of the theory in other gauges. This could be very interesting in terms of M-theory.

5 Closing Remarks

I have described the following established facts about 2T-physics

- 2T-physics, with local $\text{Sp}(2, \mathbb{R})$ & generalizations, gives emergent dynamics/spacetimes via $d+2 \rightarrow d$ holography.
- 1T-physics corresponds to d -dimensional holographs of the $d+2$ theory. Various holographs are dual; and the duality group is $\text{Sp}(2, \mathbb{R})$ & generalizations.
- When $d+2$ is in flat space each holograph has hidden $\text{SO}(d, 2)$ symmetry. Its quantum Hilbert space forms a basis for same eigenvalues of the Casimirs of $\text{SO}(d, 2)$. This applies with or without spin or supersymmetry.
- Twistor space is a particular hologram of the $d+2$ theory. In the twistor gauge the $\text{SO}(d, 2)$ becomes more manifest as compared to other holograms.
- The $10+2$ twistor string, and its particle version show hidden $(10, 2)$ holographic structures in $\text{AdS}_5 \times \text{S}^5$ superstring and supergravity. These are strong indications that other aspects of M theory also have a 2T description. See related remarks in [9] that connect to [27][29].

At this point it is hard to resist to also make some speculations, as follows

- The currently known corners of M-theory are very likely holograms of the same nature. The known M-dualities appear to be analogs of the $\text{Sp}(2, \mathbb{R})$ & generalizations. This provides hints for the underlying gauge symmetry.
- Together with the earlier indications described in the introduction, it seems now even more likely that M-theory would be most clearly formulated as a 13D theory with signature $(11, 2)$ and global supersymmetry $\text{OSp}(1|64)$.

6 Appendix: twistor string with $\text{SO}(3, 1)$ signature

Many people in this conference raised questions on the analytic continuation of $\text{SO}(3, 1)$ to $\text{SO}(2, 2)$ in the twistor superstring. Since I gave some thought to this point, I outline below what the differences are when one uses the correct signature $\text{SO}(3, 1)$. There are definite *changes in the formulation of the theory*, beyond the naive analytic continuation that inserts i in appropriate places, as follows. I work in the Berkovits formulation as it appears in [8], since this is the form of the 2T theory when we gauge fix the 2T superstring to the twistor gauge [9].

When the signature is $(3, 1)$ the twistor Y^A in [8] must be the complex conjugate of the twistor Z_A , except for the metric, namely $Y^A = \bar{Z}^A$ as seen in Eq. (14). Therefore, Z_A and \bar{Z}^A must have the same worldsheet conformal dimension $1/2$. This differs from Berkovits's $\dim(Z) = 0$ and $\dim(Y) = 1$. There is a definite consequence: There must be a shift in the stress tensor T , and in all the dimensions of the wavefunctions of physical vertices, as remarked in [9]. The following table gives the shifts

signature	(2, 2)	(3, 1)
stress tensor dimensions	$T = Y^A \partial Z_A$ 2 1 1 0	$\tilde{T} = \frac{1}{2} \bar{Z}^A \partial Z_A - \frac{1}{2} \partial \bar{Z}^A Z_A$ 2 1/2 1 1/2
SYM vertex op dimensions	$V_\phi = j^a \phi_a(Z)$ 1 1 0	$V_\phi = j^a \phi_a(Z)$ no changes 1 1 0 $Z \leftrightarrow tZ$ dim 1/2 OK
Conf. SUGRA helicity +2, dims	$V_f = Y^A f_A(Z)$ 1 1 0	$V_f = Z^A f_A(Z)$ 1 1/2 1/2 $Z \leftrightarrow tZ$
Conf. SUGRA helicity -2, dims	$V_g = \partial Z_A g^A(Z)$ 1 1 0 0	$V_g = \partial Z_A g^A(Z)$ 1 1 1/2 -1/2 $Z \leftrightarrow tZ$
Amplitudes instanton number	$T_n = T + n \partial(YZ)$ n	$\tilde{T}_n = \tilde{T} + n \partial(\bar{Z}Z)$ $= T_n - \frac{1}{2} \partial(YZ) \leftrightarrow n \rightarrow n - 1/2$

Note that the SYM vertex $\phi_a(Z)$ has still dimension zero, but now it must be constructed from Z that has dimension 1/2 instead of Z that has dimension 0. However since $\phi_a(Z)$ is homogeneous it means it is constructed from ratios of components of Z_A . Therefore the same wavefunctions will still appear, and so it seems that nothing changes in the SYM sector. This is good news.

Although the SYM wavefunction $\phi_a(Z)$ has the same dimension for either $SO(3, 1)$ or $SO(2, 2)$, the conformal supergravity wavefunctions $f_A(Z), g^A(Z)$ must have different dimensions, as shown in the table. I had hoped that $f_A(Z)$ with $\dim(f) = \frac{1}{2}$ and $g^A(Z)$ with $\dim(g) = -\frac{1}{2}$ would not exist (and therefore get rid of the “conformal gravity pollution” in the theory); but apparently they do exist without much modification also.

Another change is the stress tensor itself. The shift is equivalent to a shift in the instanton number $n \rightarrow n - 1/2$. I have not checked the details if this is cancelled by additional modifications, and whether this is harmless as well.

References

- [1] I. Bars, C. Deliduman and O. Andreev, “Gauged Duality, Conformal Symmetry and Spacetime with Two Times”, Phys. Rev. **D58** (1998) 066004 [arXiv:hep-th/9803188]. For reviews of subsequent work see: I. Bars, “Two-Time Physics”, in the Proc. of the 22nd Intl. Colloq. on Group Theoretical Methods in Physics, Eds. S. Corney et. al., World Scientific 1999, [arXiv:hep-th/9809034]; “Survey of two-time physics,” Class. Quant. Grav. **18**, 3113 (2001) [arXiv:hep-th/0008164]; “2T-physics 2001,” AIP Conf. Proc. **589** (2001), pp.18-30; AIP Conf. Proc. **607** (2001), pp.17-29 [arXiv:hep-th/0106021].
- [2] I. Bars, “2T physics formulation of superconformal dynamics relating to twistors and supertwistors,” Phys. Lett. B **483**, 248 (2000) [arXiv:hep-th/0004090].
- [3] I. Bars and M. Picon, in preparation.
- [4] E. Witten, “Perturbative gauge theory as a string theory in twistor space”, Commun. Math. Phys. **252** (2004) 189 [arXiv:hep-th/0312171]; “Parity invariance for strings in twistor space”, hep-th/0403199.

- [5] F. Cachazo, P. Svrcek and E. Witten, “ MHV vertices and tree amplitudes in gauge theory”, JHEP **0409** (2004) 006 [arXiv:hep-th/0403047]; “ Twistor space structure of one-loop amplitudes in gauge theory”, JHEP **0410** (2004) 074 [arXiv:hep-th/0406177]; “Gauge theory amplitudes in twistor space and holomorphic anomaly”, JHEP **0410** (2004) 077 [arXiv:hep-th/0409245]
- [6] N. Berkovits, “An Alternative string theory in twistor space for N=4 Super Yang-Mills”, Phys. Rev. Lett. **93** (2004) 011601 [arXiv:hep-th/0402045].
- [7] N. Berkovits and L. Motl, “Cubic twistorial string field theory”, JHEP **0404** (2004) 56 [arXiv:hep-th/0403187].
- [8] N. Berkovits and E. Witten, “Conformal supergravity in twistor-string theory”, JHEP **0408** (2004) 009 [arXiv:hep-th/0406051].
- [9] I. Bars, “Twistor superstring in 2T-physics,” Phys. Rev. **D70** (2004) 104022 [arXiv:hep-th/0407239].
- [10] I. Bars, “ Hidden 12-dimensional structures in $AdS_5 \times S^5$ and $M^4 \times R^6$ supergravities,” Phys. Rev. D **66**, 105024 (2002) [arXiv:hep-th/0208012].
- [11] I. Bars, “ A mysterious zero in $AdS(5) \times S^5$ supergravity,” Phys. Rev. D **66**, 105023 (2002) [arXiv:hep-th/0205194].
- [12] I. Bars and M. Günaydin, Comm. Math. Phys. **91** (1983) 31.
- [13] M. Günaydin, P. van Nieuwenhuizen, N.P. Warner, Nucl. Phys. **B255** (1985) 63.
- [14] E. Witten, “Conformal field theory in four and six dimensions”, In *Oxford 2002, Topology, geometry and quantum field theory*, p.405.
- [15] I. Bars, “Duality and Hidden Dimensions”, hep-th/9604200, published in *Frontiers in Quantum Field Theory*, Eds. H. Itoyama et. al., World Scientific (1996), page 52. See also, “Supersymmetry, P-Brane Duality And Hidden Space-Time Dimensions”, Phys. Rev. **D54** (1996) 5203 [=hep-th/9604139].
- [16] I. Bars, S-theory, Phys. Rev. **D55** (1997) 2373 [arXiv:hep-th/9607112]; “Algebraic structure of S-theory”, hep-th/9608061; “Black hole entropy reveals a twelfth dimension”, Phys. Rev. D **55** (1997) 3633 [arXiv:hep-th/9610074]; “A Case for fourteen dimensions”, Phys. Lett. **B403** (1997) 257 [arXiv:hep-th/9704054]
- [17] C. Vafa, “Evidence for F-Theory”, Nucl. Phys. **B469** (1996) 403 [arXiv:hep-th/9602022]
- [18] I. Bars and C. Kounnas, “Theories with two times”, Phys. Lett. **B402** (1997) 25 [arXiv:hep-th/9703060]; “String and particle with two times”, Phys. Rev. **D56** (1997) 3664 [arXiv:hep-th/9705205]; I. Bars and C. Deliduman, “Superstrings with new supersymmetry in (9,2)-dimensions and (10,2)-dimensions”, Phys. Rev. **D56** (1997) 6579, [arXiv:hep-th/9707215]; “Gauge principles for multi - superparticles”, Phys. Lett. **B417** (1998) 240 [arXiv:hep-th/9710066]

- [19] I. Bars, C. Deliduman and D. Minic, “ Lifting M-theory to Two-Time Physics”, Phys. Lett. **B457** (1999) 275 [arXiv:hep-th/9904063].
- [20] I. Bars, “Toy M-model”, unpublished. For a brief outline of the model see [19][2] and third reference in [1].
- [21] I. Bars, “Conformal symmetry and duality between free particle, H-atom and harmonic oscillator”, Phys. Rev. **D58** (1998) 066006 [arXiv:hep-th/9804028]; “Hidden Symmetries, $AdS_d \times S^n$, and the lifting of one-time physics to two-time physics”, Phys. Rev. **D59** (1999) 045019 [arXiv:hep-th/9810025].
- [22] I. Bars, “Two time physics with gravitational and gauge field backgrounds”, Phys. Rev. **D62**, 085015 (2000) [arXiv:hep-th/0002140]; I. Bars and C. Deliduman, “ High spin gauge fields and two time physics”, Phys. Rev. **D64**, 045004 (2001) [arXiv:hep-th/0103042].
- [23] I. Bars and C. Deliduman, “Gauge symmetry in phase space with spin: a basis for conformal symmetry and duality among many interactions”, Phys. Rev. **D58** (1998) 106004, [arXiv:hep-th/9806085]. For additional quantum properties of the spinning theory in covariant quantization, including interactions, see the first reference in [26].
- [24] I. Bars, C. Deliduman and D. Minic, “ Supersymmetric two-time physics” , Phys. Rev. **D59** (1999) 125004 [arXiv:hep-th/9812161].
- [25] I. Bars, C. Deliduman and D. Minic, “ Strings, Branes and Two-Time Physics” , Phys. Lett. **B466** (1999) 135 [arXiv:hep-th/9906223].
- [26] I. Bars, “ Two-time physics in Field Theory” , Phys. Rev. D **62**, 046007 (2000), [arXiv:hep-th/0003100]; “ $U^*(1,1)$ noncommutative gauge theory as the foundation of 2T-physics in field theory,” Phys. Rev. D **64**, 126001 (2001) [arXiv:hep-th/0106013]; I. Bars and S. J. Rey, “ Noncommutative $Sp(2,R)$ gauge theories: A field theory approach to two-time physics,” Phys. Rev. D **64**, 046005 (2001) [arXiv:hep-th/0104135].
- [27] M.A. Vasiliev, “ Higher spin superalgebras in any dimension and their representations” , arXiv:hep-th/0404124.
- [28] I. Bars, “ Stringy Evidence for D=11 Structure in Strongly Coupled Type IIA Superstring” , Phys.Rev. **D52** (1995) 3567 [arXiv:hep-th/9503228].
- [29] M. Bianchi, J. F. Morales and H. Samtleben, “On stringy $AdS(5) \times S^{*5}$ and higher spin holography,” JHEP **0307** (2003) 062 [arXiv:hep-th/0305052]; N. Beisert, M. Bianchi, J. F. Morales and H. Samtleben, “On the spectrum of AdS/CFT beyond supergravity,” JHEP **0402** (2004) 001 [arXiv:hep-th/0310292]; “Higher spin symmetry and $N = 4$ SYM,” [arXiv:hep-th/0405057].